Optimal Arbitrage with Limit Orders in Day-Ahead Electricity Markets

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Overview of the study

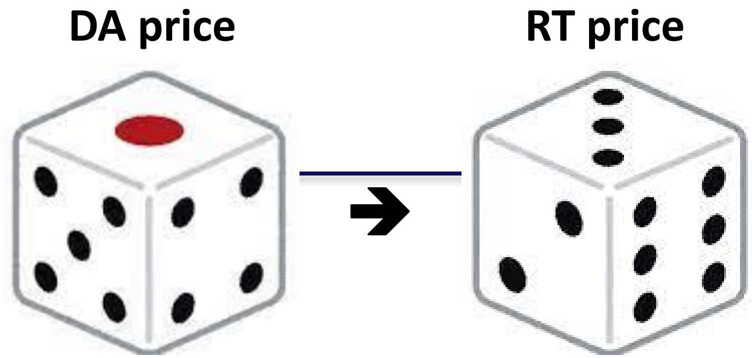
- Numerous studies focus on intermarket strategies.
- **Traditional focus**: Which market to trade in and when (market order strategies; MO).
- **Our study:** Determines the optimal price to bid (limit order strategy; LO).
- Formulate and empirically test the relationship between arbitrage profit, market volatility, and forward premiums

Studies dealing with arbitrage (intermarket trading)

DA	ID	RT	Country	Study
✓	✓		Germany and Poland	Maciejowska et al. (2019)
✓		✓	Netherlands	Boogert and Dupont (2005)
✓		✓	California	Borenstein et al. (2008)
✓	✓	✓	Germany and Poland	Janczura and Wojcik (2022)
	✓		Germany	Kath and Ziel (2018)
	✓	✓	GB	Pozzetti and Cartlidge (2020)
	✓	✓	Austria	Bunn and Kermer (2021)
	✓	✓	Japan	Matsumoto et al. 2021)
	✓	✓	Netherlands	Demir et al. (2022)
	✓	✓	Netherlands	Demir et al. (2023)
✓			Japan	Matsumoto and Endo (2021)
✓		✓	US	Hogan (2016)
✓		✓	US	Hadsell (2007)
✓		✓	US	Li et al. (2015)
✓		✓	US	Baltaoglu et al. (2018)

All "**market order**" strategies (*US case using LMP is a different problem setting)

Concept of limit order strategy



◆ Two-stage Game Concept (e.g. of dice):

- Electricity pricing: DA (day-ahead) and RT (real-time)
- Prices determined like dice rolls (range of 1 to 6)

◆ Initial Perception:

- No arbitrage opportunities; expected values of DA and RT are 3.5.

◆ Reality with Limit Order Trades:

- Arbitrage opportunities exist by setting buy & sell limit orders at 3.5.
- Expected profits of 1.5 can be achieved (∵{1,2,3}→buy; {4,5,6}→sell).

◆ Suggestion:

- Uniform auction markets have optimal bid prices for profit.
- Arbitrage profits higher through limit orders than market orders.

Arbitrage profit for bidding price

- Arbitrage profit $\pi(z)$ when simultaneous bids to sell and buy in the day-ahead market at price z is calculated as follows (assuming a normal distribution for DA).

$$\pi(z) = \int_{-\infty}^{z} \{E[RT|DA] - DA\} f(DA)\,dDA + \int_{z}^{\infty} \{DA - E[RT|DA]\} f(DA)\,dDA$$

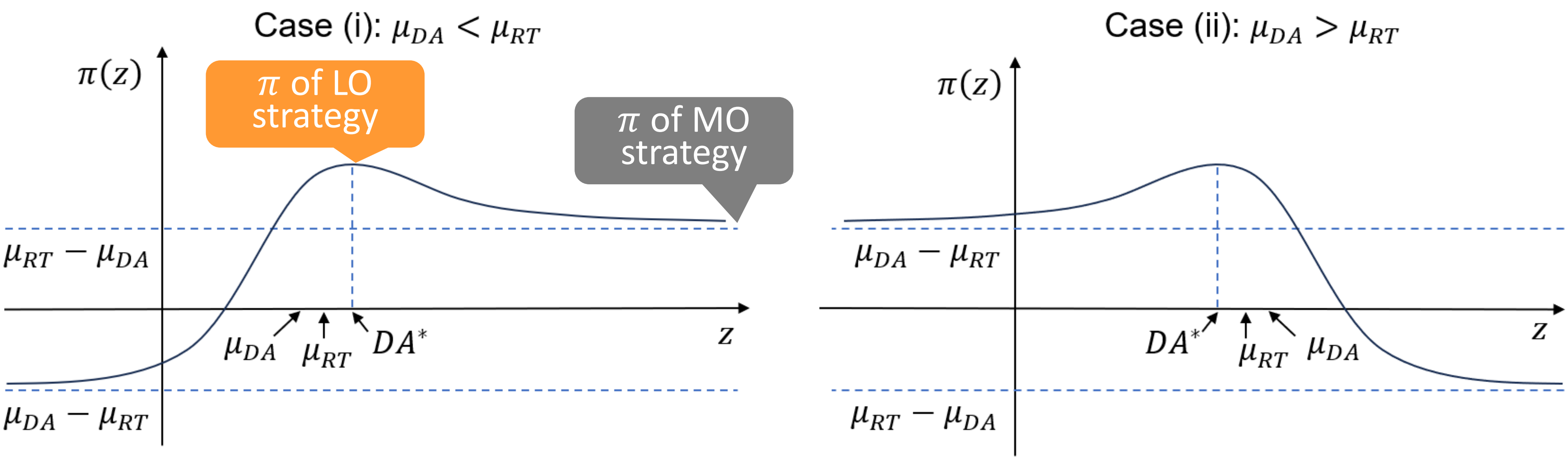
$$= (1-\alpha)\{2\sigma\phi(z') - 2\mu'\Phi(z') + \mu'\}$$

Forward premium: $\mu := \mu_{DA} - \mu_{RT}$

Where $z' := \frac{z-\mu_{DA}}{\sigma}$, $\mu' := \frac{\mu}{1-\alpha}$, $f(DA|\mu_{DA}, \sigma) = \frac{1}{\sigma}\phi\left(\frac{DA-\mu_{DA}}{\sigma}\right)$

$E[RT|DA] = \alpha DA + (\mu_{RT} - \alpha\mu_{DA})$

Stdev. of DA (σ)



Relationship w/ DA-moments of optimal arbitrage profit

- Optimal arbitrage profit π^* is monotonically increasing with both σ and $|\mu|$
- π^* converges to 0 when both σ and $|\mu|$ are 0
- Even when forward premium is 0, profit from limit orders grows in proportion to σ

Stdev. of DA | Forward premium

$$\pi^* = |\mu| + 2\sigma(1-\alpha)\int_{-\infty}^{-\frac{|\mu|}{\sigma(1-\alpha)}} \Phi(x)\,dx \qquad \mu := \mu_{DA} - \mu_{RT}$$

For Std of DA: σ			
σ	0	...	∞
$\pi^{*\prime}(\sigma)$	0	$+$	$\sqrt{\frac{2}{\pi}}(1-\alpha)$
$\pi^{*\prime\prime}(\sigma)$	0	$+$	0
π^*	$\lvert\mu\rvert$	↗	∞

For Absolute Forward premium $\lvert\mu\rvert$			
$\lvert\mu\rvert$	0	...	∞
$\pi^{*\prime}(\lvert\mu\rvert)$	0	$+$	1
$\pi^{*\prime\prime}(\lvert\mu\rvert)$	0	$+$	0
π^*	$\sqrt{\frac{2}{\pi}}(1-\alpha)\sigma$	↗	∞

Forecast models

- In comparing arbitrage profits, we combine 5 trading strategies (2 MO, 3 LO), 5 model formulas, and 4 statistical methods.

Strategy — **Model formula** — **Stat. model**

MO strategy:
- MO_each → $DA_t = \boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{X}_{\boldsymbol{DA,t}} + \varepsilon_t$; $RT_t = \boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{X}_{\boldsymbol{RT,t}} + \varepsilon_t$
- MO_diff → $Diff_t = \boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{X}_{\boldsymbol{Diff,t}} + \varepsilon_t$

LO strategy:
- LO_pred → $RT_t = \boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{X}_{\boldsymbol{RT,t}} + \varepsilon_t$
- LO_reg1 → $Diff_t = \alpha_1 DA_t + \boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{X}_{\boldsymbol{Diff,t}} + \varepsilon_t$
- LO_reg2 → $RT_t = \alpha_2 DA_t + \boldsymbol{\beta}^{\mathrm{T}} \boldsymbol{X}_{\boldsymbol{RT,t}} + \varepsilon_t$

Stat. model:
- OLS
- LASSO
- Ridge
- QR
- naive
  price at the same hour of the previous day

(LASSO, Ridge: 5-fold CV)

, where $\boldsymbol{X}_{\boldsymbol{RT,t}} := [RT_{t-1}, RT_{t-7}, Saturday_t, Holiday_t, Gas_t, \boldsymbol{Forecast_t}]$

weather forecast (JP) or demand/renewable forecast (GB)

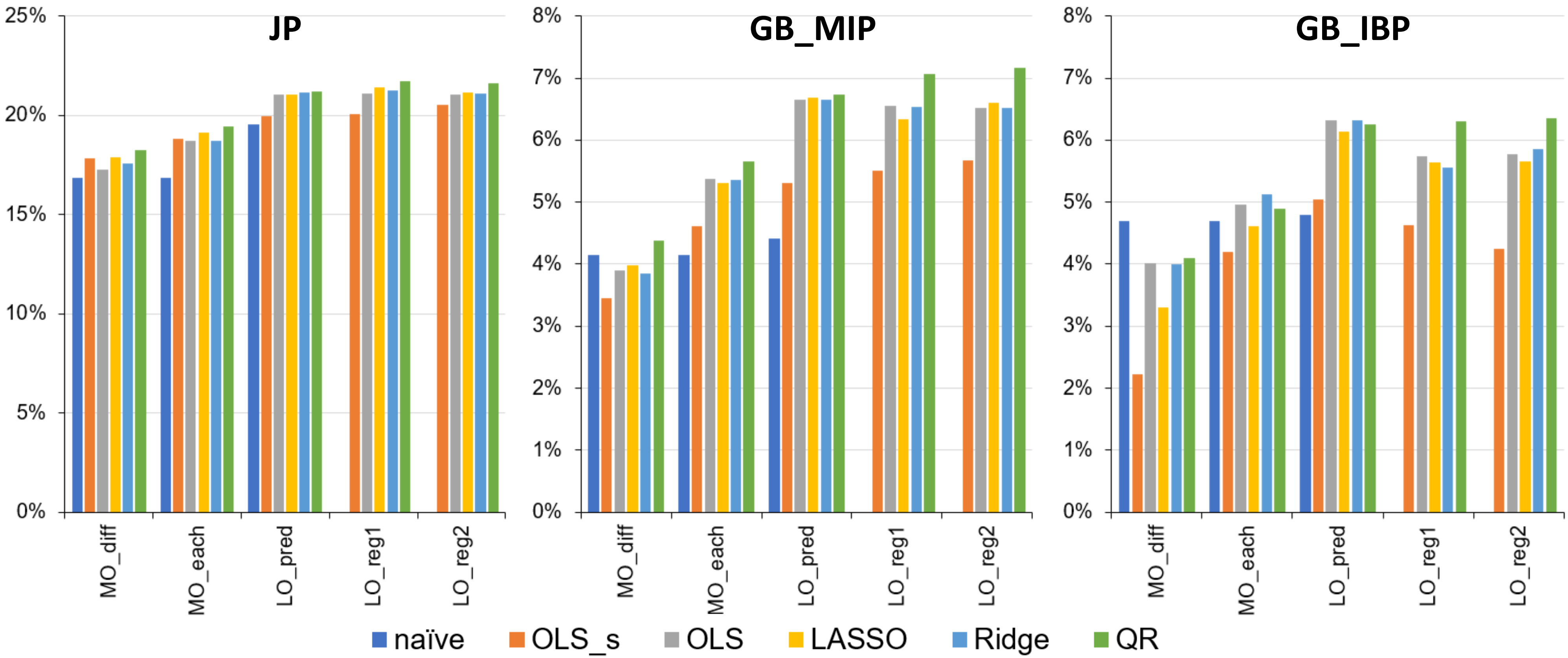
Used data for back test

◆ Studied 3 cases: JP (DA vs. IBP), GB_MIP (DA vs. MIP), and GB_IBP (DA vs. IBP)

	JP (Japan)	GB (Great Britain)
In sample	84 days	365 days
Out of sample (daily rolling)	Jun. 1, 2022, to Dec. 31, 2022.	Jan. 1, 2021, to Dec. 31, 2022.
Granularity	48 time data par day (half hourly data)	24 time data par day (hourly data)
Spot price	Tokyo price (JEPX)	GB price (Refinitiv Eikon)
Imbalance price (intraday price)	Tokyo price (Imbalance Price: IBP)	“Imbalance Price (IBP)” and “Market Index Price (MIP)" published (Elexon BMRS)
Gas price	N/A	NBP gas price (Refinitiv Eikon)
Forecast values	Max. temp. and prob. of rain forecast (by the JMA)	Load, Wind and Solar power, and De-rated Margin forecast (day-ahead forecasts by Elexon)

Arbitrage profits (main result)

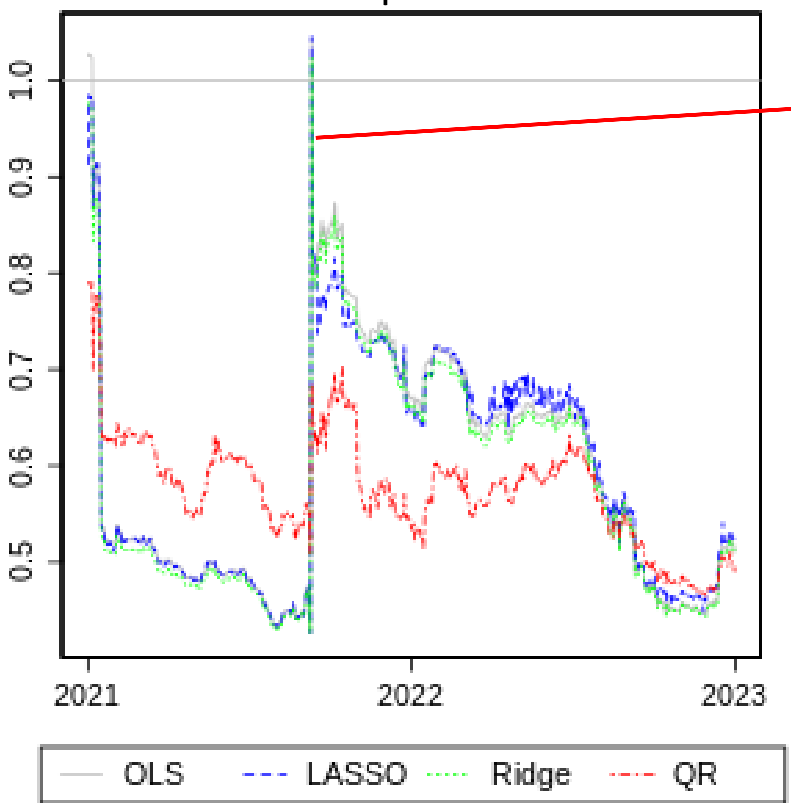
- Arbitrage profits are generally larger for LO strategies than MO.
- JP is more profitable than GB at baseline (i.e., MO), suggesting the market inefficiencies.
- For the GB market, the QR model is the most profitable, presumably due to the robustness and accuracy of the model (see later)



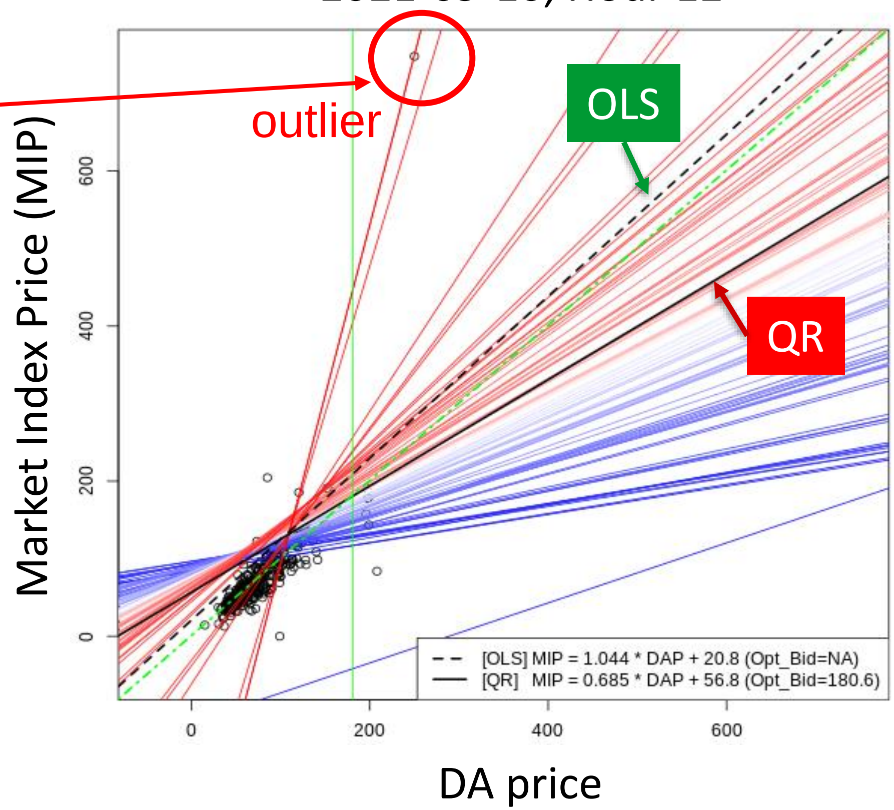
Estimated result of α

- In OLS, LASSO, and Ridge regression, the slope α is extremely sensitive to outlier.
- In contrast, QR, which can be applied to any probability distribution, has stable results for estimating α.

Estimated slope α for Hour 12



2021-09-10, Hour 12



Summary

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- Proposed a trading method using limit orders and uncovers **arbitrage opportunities in price volatility**, beyond just forward premium.
- Provided valuable insights into market inefficiencies and the forward premiums.

Result and suggestion

- **LO strategies have higher expected profits** than MO strategies.
  - **QR model**, adept at handling skewed density, **offers superior profitability**.
- Arbitrage profits can be interpreted as **compensates for reducing** not only market price differences but also **forward price volatility**.
- The method benefits renewable business as well as financial players.