Volatility in Solar Renewable Energy Certificates: Jumps and Fat Tails

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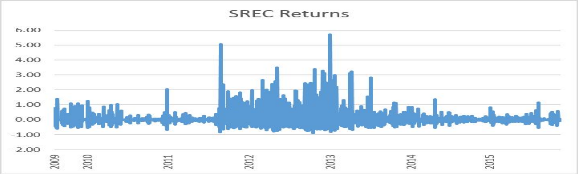
Increasing role for rooftop solar

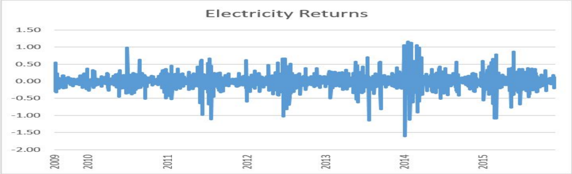
- interest in expanding renewable energy
  - ⇒ attention to "distributed solar"
    - solar panels on household rooftops
    - aka "rooftop solar"
  - ▷ potentially costly investment
  - ▷ motivates subsidy schemes to encourage installation
- an important class of such schemes entails "solar credits" (SREC)
  - ▷ provides potential stream of future benefits from solar installation
    - provides future payments based on "credit price"
    - in addition to any cost savings from defrayed electricity expenditures
  - ▷ became popular in US in last decade
  - ▷ one important example: New Jersey
    - one of oldest, largest solar markets in US
    - second in size only to California

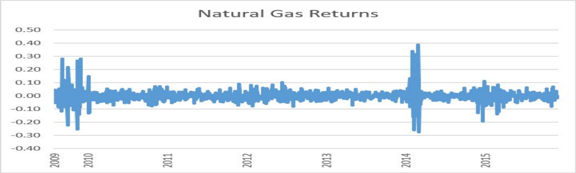
(solar) investment under uncertainty

- rooftop solar has key characteristics of “investment under uncertainty” problem
  - significant, irreversible up-front investment
  - benefits from investment depend on stochastic variable
    - possibly more than one (SREC, electricity)
- how does the variable of interest evolve?
  - GBM?
  - time-varying volatility?
  - jumps?
- motivates empirical investigation into properties of key variables

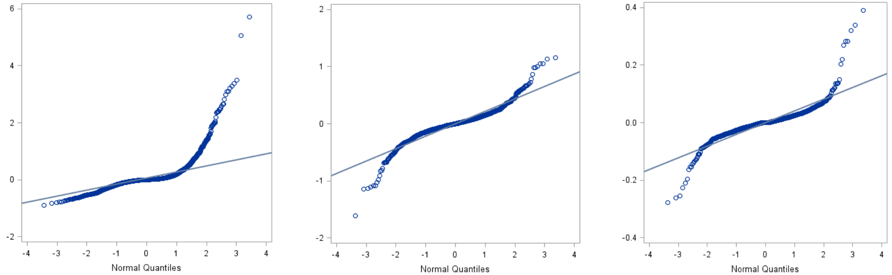
Price returns: SREC (T), electricity (M), natural gas (B)







QQ plots: SREC (L), electricity (C), natural gas (R)



Price Discontinuities

- denote spot price at t by P_t
  - SREC
  - electricity
  - natural gas
- if P_t follows geometric Brownian motion then $x_t \equiv \ln(P_t/P_{t-1})$ follows Brownian motion

$$x_t = \mu + \sigma z_t, \text{ where } z_t \text{ is increment of standard Wiener process}$$

- suppose “jumps” arrive infrequently, modeled as Poisson process
- jump occuring during interval $[t, t+\Delta t]$ is

$$dn_t = \begin{cases} 0 & \text{with probability} \quad 1-\lambda dt \\ 1 & \text{with probability} \quad \lambda dt \end{cases}$$

Data Generation Mechanism

JD The mixed jump-diffusion process is given by

$$x_t = \mu + \sigma z_t + J_t$$

GPD adapting the PD process to allow for time-varying volatility, via a GARCH(1,1) framework, yields

$$x_t = \mu + \sqrt{h_t} z_t, \quad \text{where}$$
$$h_t = \kappa + \alpha (x_t - \mu)^2 + \beta h_{t-1}$$

GJD adapting the JD process to allow for time-varying volatility, via a GARCH(1,1) framework, yields

$$x_t = \mu + \sqrt{h_t} z_t + J_t, \quad \text{where}$$
$$h_t = \kappa + \alpha (x_t - \mu)^2 + \beta h_{t-1}$$

Maximum likelihood estimation

- we proceed by maximizing the log-likelihood function:

$$L(\phi; x_t) = -T\lambda - \frac{T}{2}\ln(2\pi) + \sum \ln\left[\sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \frac{1}{\sqrt{h_t + n\delta^2}} \exp\left(\frac{-(x_t - \mu - n\theta)}{2(h_t + n\delta^2)}\right)\right]$$

by choice of the parameter vector $(\mu, \kappa, \alpha, \beta, \lambda, \theta, \delta)$

- this representation subsumes the four possible stochastic processes

PD: $\lambda = 0; h_t = \sigma^2$

JD: $\lambda > 0; h_t = \sigma^2$

GPD: $\lambda = 0; h_t = \kappa + \alpha(x_t - \mu)^2 + \beta h_{t-1}$

GJD: $\lambda > 0; h_t = \kappa + \alpha(x_t - \mu)^2 + \beta h_{t-1}$

Data and its properties: summary statistics

Variable	Solar Returns	Natural Gas Returns	Electricity Returns
Sample Range			
Start	Aug 01, 2009	Aug 01, 2009	Aug 01, 2009
End	Nov 30, 2015	Nov 30, 2015	Nov 30, 2015
Summary Statistics			
Mean	0.0648	-0.00029	-0.0003
Median	-0.0014	0.00000	0.00000
Minimum	-0.8867	-0.27844	-1.6136
Maximum	5.7102	0.39007	1.1624
Variance	0.2137	0.00166	0.0471
Std. Dev.	0.4623	0.04071	0.2171
Coeff. of Variation	713.6	-14,172.5	-84,591.0
Skewness	4.1873	1.11170	-0.3023
Kurtosis	29.4956	20.22889	7.6107
n	2099	1599	1606
Test of Normality			
Kolmogoro Smirnov	0.2508	0.1133	0.1017
p-value	<0.01	<0.01	<0.01
Unit Root Test			
Modified Dickey Fuller	-4.806	-4.909	-4.446
1% critical-value†	-3.48	-3.48	-3.48
Lags	20	23	24

Note: Statistics are based on a sample of daily observations. Solar returns are based on SREC prices; natural gas returns based on Henry Hub spot prices; and electricity returns are based on PJM prices.

†: Critical values are based on Elliot et al. (1996).

Log-likelihood test results

Variable	$LR_{PD,JD}$	$LR_{PD,GPD}$	$LR_{JD,GJD}$	$LR_{GPD,GJD}$
Solar Credits (*SREC*)	2539.02	1714.60	570.62	1395.04
	(0.000)	(0.000)	(0.000)	(0.000)
Natural Gas (*HH*)	812.45	999.34	254.05	67.16
	(0.000)	(0.000)	(0.000)	(0.000)
Electricity (*PJM*)	540.64	634.21	246.59	153.02
	(0.000)	(0.000)	(0.000)	(0.000)

Note: Based on a sample of 2009 observations. *p-values* are given in the parentheses.

Log-likelihood estimation results

Variable	μ	κ	α_1	β_1	λ	θ	δ
Solar Credits	0.0017	0.0007***	0.3899***	0.3216***	0.2065***	0.3818***	0.6744***
	(0.001)	(0.0001)	(0.058)	(0.050)	(0.019)	(0.056)	(0.045)
Electricity Prices	-0.8772	7.9596**	0.2219***	0.6945***	0.1654***	10.3549***	21.7218***
	(0.490)	(5.520)	(0.027)	(0.032)	(0.062)	(2.836)	(4.758)
Natural Gas	-0.0704	0.5013***	0.1160***	0.8010***	0.0195***	0.9577 ***	10.3349***
	(0.062)	(0.105)	(0.016)	(0.024)	(0.007)	(2.77)	(3.029)

The standard errors are presented in the parentheses. σ not estimated in GJD model.
***, **, and * indicate statistical significance at the 1%, 5% and 10% levels, respectively.

Summary of results

- Examined SREC, electricity and natural gas prices prices over the same time period
- Data does not support conventional Brownian motion process
- Model fit improved by allowing for jumps, GARCH
- Characteristic applies for SREC, electricity and natural gas prices
  - fat tails
  - implication for investment under uncertainty?
- estimated jump probability (λ) noticeably lower for natural gas than electricity or SREC
  - related to market (im)maturity?
    - NG markets deregulated by late 1990s
    - PJM electricity market shorter experience with derregulation
    - SREC market newest of all three

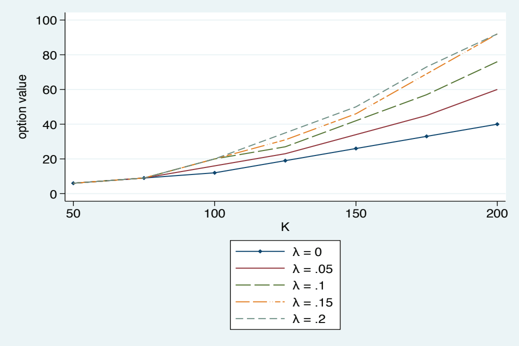
Implication of jumps?

- consider "investment under uncertainty" problem
  - irreversible upfront installation cost
  - benefits are stochastically varying
    - SREC
    - electricity
  - standard 'Dixit-Pindyck' model finds "option value" from delaying investment
    - based on GBM (stochastic) variable
    - manifests as increase in critical value from investing that would trigger action
  - how does the possibility of jumps or fat tails influence this understanding?

Numerical results

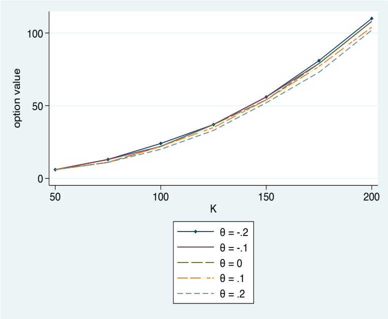
- determining optimal investment rule under GBM is relatively straightforward
- allowing for jumps greatly complicates problem
  - requires use of numerical techniques
    - we determine the cutoff vale V^* that triggers investment
    - option value (OV) reflects expected time to reach V^*
    - larger $V^* \rightarrow$ larger OV
  - numerical results depicting implications of various comparative dynamics:
    - increasing jump probability
    - increasing mean jump value
    - increasing variance in jump value

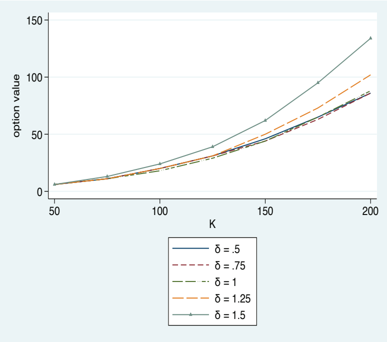
Influence of jump probability



- for small levels of investment, influence of jump probability is negligible
- past some point, potential for jumps delays investment
- at larger investment costs, this effect can be pronounced

Implication of increasing moments of jump process?





- impact on investment by an increase in
  - mean of jump values: (very) modest decrease in OV of waiting
    - → small acceleration in investment
  - variance of jump values: increase in OV of waiting → decrease
    - → delay in investment